Data Representation With Optimal Transport

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Finding a useful mathematical model to represent data (functions, signals, images) can facilitate the solution of numerous problems including signal or image detection, estimation, filtering, reconstruction, compression, system identification and classification. Fourier transforms, for example, render convolution operations into multiplications in Fourier domain, thereby simplifying the solution of linear shift-invariant systems. Wavelet transforms, on the other hand, are well-suited for detecting and analyzing signal transients at different resolutions. Given the emergence of modern machine learning problems such as signal and image classification, the demand for nonlinear signal and image representation techniques has significantly increased.

Recently nonlinear representation techniques using the mathematics of optimal transport have emerged. The idea is based on using optimal transport theory to establish a unique relation (a transport map) between a fixed function (or measure), and any function (measure) one wishes to represent. The unique transport map that pushes the reference function onto the function of interest can be interpreted as a new representation for data (functions, signals, and images). Like Fourier and Wavelet transforms, this emerging transport-based representation is invertible, and has interesting mathematical properties that can help "convexify" data classes. Therefore, they are able render signal processing problems that are difficult (e.g. nonlinear) to solve in signal domain simpler (e.g. linear) to solve in transport representation domain. This talk will highlight this emerging mathematical representation technique, describing numerous signal and image analysis applications along the way.